Starter Questions

Simplify fully $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$

¿ $\frac{2}{5}x^{-2}$

Express 8^{2x+3} in the form 2^y, stating y in terms of x.

¿ 2^{6x+9}

Given that $y = 2^x$,

(a) express 4^x in terms of y.

¿ y^2

(b) Hence, or otherwise, solve

$$8(4^x) - 9(2^x) + 1 = 0.$$

$x = -3 \; or \; 0$

G Differentiation

G1

Understand and use the derivative of $\mathbf{f}(x)$ as the gradient of the tangent to the graph of $y = \mathbf{f}(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second derivatives; differentiation from first principles for small positive integer powers of x and for $\sin x$ and $\cos x$

Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves and points of inflection.

Students should be able to:

- recognise and use the notations $\mathrm{f}'(x)$, $\frac{\mathrm{d}y}{\mathrm{d}x}$, $\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{f}(x))$

Tuesday, May 27, 2025

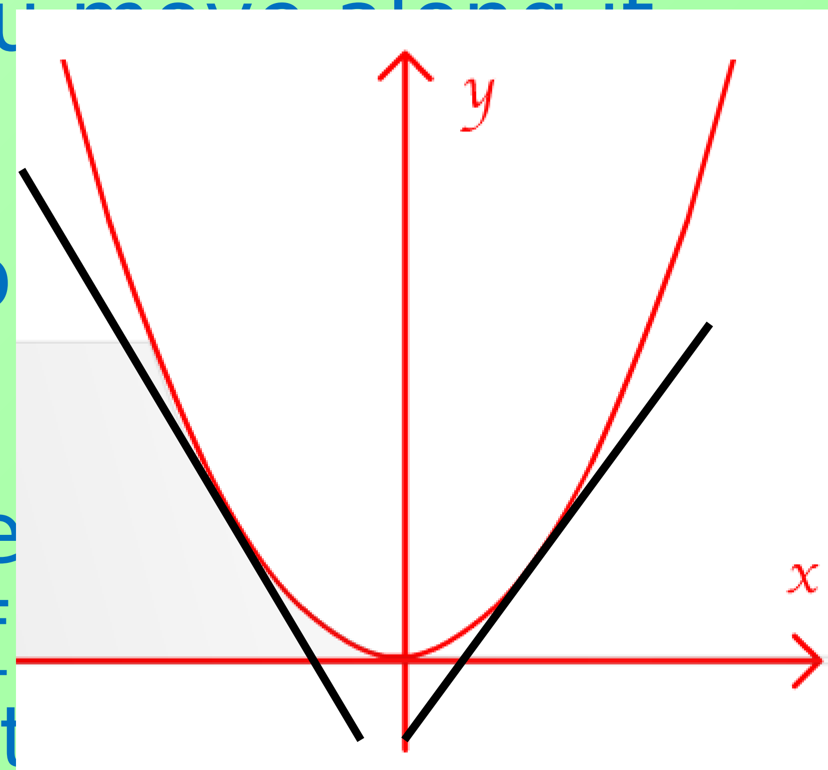
4.1 Differentiation from first principles

The gradient of a curve is how steep it is.

However, unlike a line, the gradient of a curve changes as you

You can only give the gradient at a specific p on the curve.

The value of the gradie is known as the <u>rate of</u> <u>change of with respect</u>

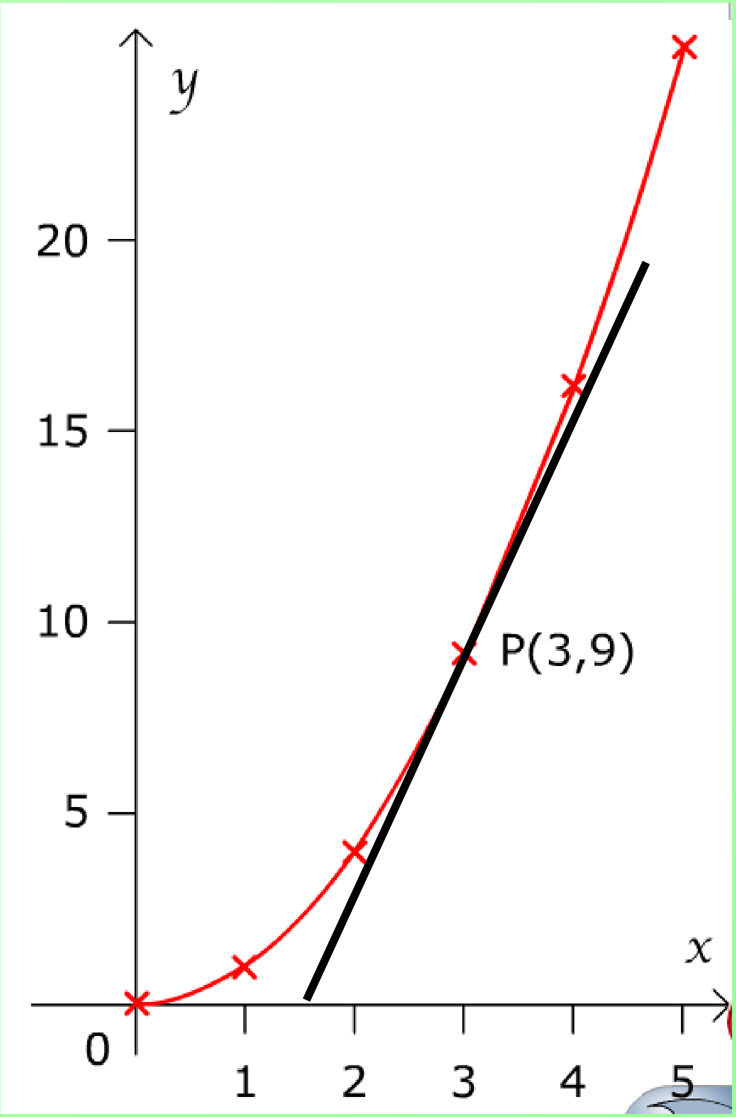


4.1 Differentiation from first principles

Differentiation from first principles is a method of finding the gradient and hence the rate of change.

Consider the gradient of the function $y = x^2$ at the point (3, 9).

We could draw a tangent and work out

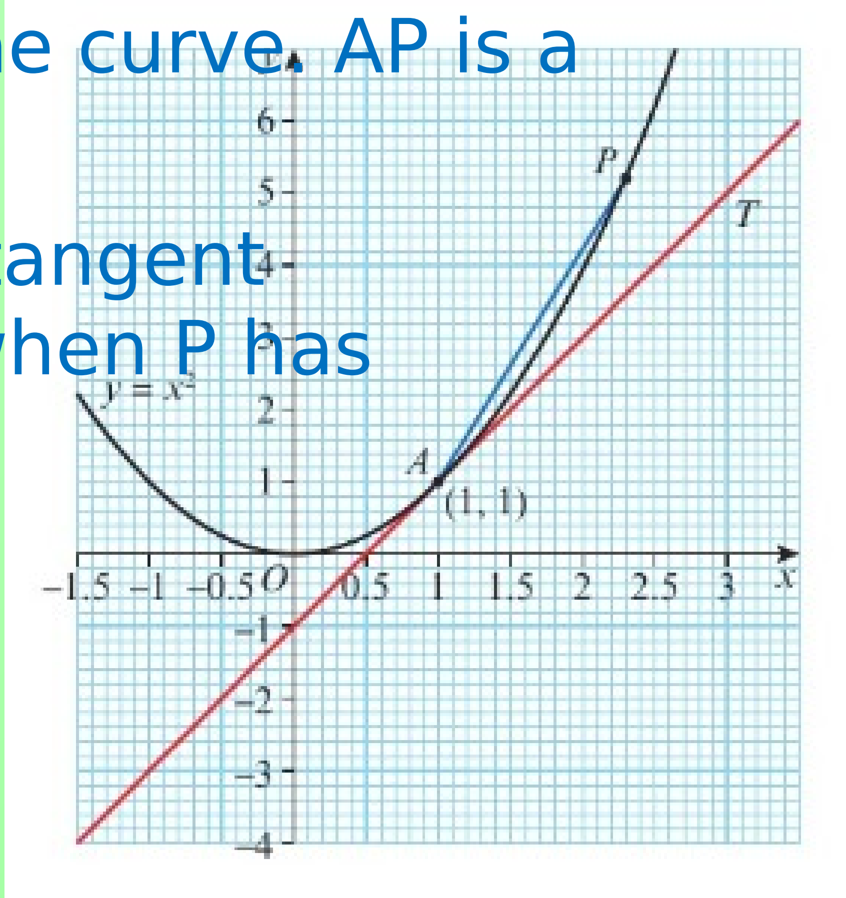


4.1 Differentiation from first principles

Example 1

Consider the graph of .
The tangent at the point A(1,1) is shown in red.
P is a point elsewhere on the curve. AP is a chord.

a) Find the gradient of the tangent
b) Find the gradient of AP when P has coordinates:



4.1 Differentiation from first principles

You try

F is the point with coordinates (3, 9) on the curve with equation $y = x^2$.

a Find the gradients of the chords joining the point F to the points with coordinates:

i (4, 16) **ii** (3.5, 12.25) **iii** (3.1, 9.61)

iv (3.01, 9.0601) **v** $(3 + h, (3 + h)^2)$

b What do you deduce about the gradient of the tangent at the point (3, 9)?

a **i** Gradient $= \dfrac{16-9}{4-3} = \dfrac{7}{1} = 7$

ii Gradient $= \dfrac{12.25-9}{3.5-3} = \dfrac{3.25}{0.5} = 6.5$

iii Gradient $= \dfrac{9.61-9}{3.1-3} = \dfrac{0.61}{0.1} = 6.1$

iv Gradient $= \dfrac{9.0601-9}{3.01-3} = \dfrac{0.0601}{0.01} = 6.01$

v Gradient $= \dfrac{(3+h)^2-9}{(3+h)-3}$

$$= \frac{6h+h^2}{h}$$

$$= \frac{h(6+h)}{h}$$

$$= 6 + h$$

b When h is small, the gradient of the chord is close to the gradient of the tangent, and $6 + h$ is close to the value 6.
So the gradient of the tangent at (3, 9) is 6.

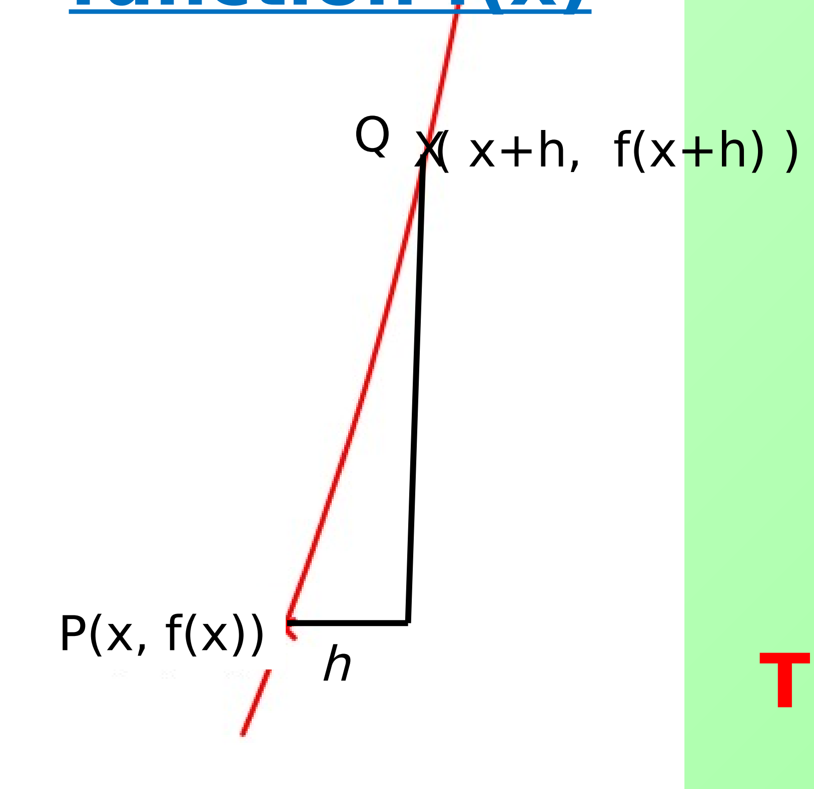
4.1 Differentiation from first principles

Watch G1-11 through to G1-15

https://sites.google.com/view/tlmaths/home/a-level-maths/as-only/g-differentiation/g1-differentiation-from-first-principles#h.318gmmuvtvhu

4.1 Differentiation from first principles

The general case for any function f(x)



We want to find the gradient of the curve, , at a general point .

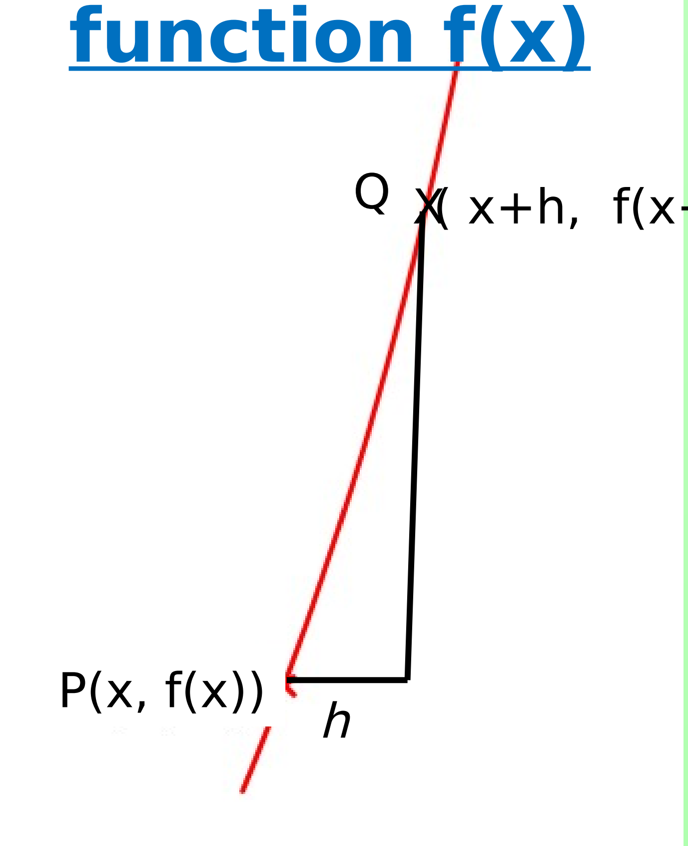
This is given by:

This is on the formula sheet

The limiting value is denoted by and is known as the derived function or the derivative of .

4.1 Differentiation from first principles

The general case for any function f(x)



We want to find the gradient of the curve, , at a general point .

This is given by:

This is on the formula sheet

You may also see the following notation: and

and

4.1 Differentiation from first principles

Example 2

Prove from first principles that the derivative of is .

Let

So

=

=

=

=

=

As ,

4.1 Differentiation from first principles

Example 3

Use differentiation from first principles to work out the derivative of $y = x^3$

Let

So

=

=

=

=

=

As ,

4.1 Differentiation from first principles

You try…

Differentiate, from first principles, $y = 3x^2$

Let $f(x) \quad = 3x^2$

So $f(x + h) \quad = 3(x + h)^2$

Now $f'(x) \quad =$

$=$

$=$

$=$

$=$

$=$

As ,

4.1 Differentiation from first principles

Example 4

Differentiate, from first principles, $y = 5$

Let $f(x) = 5$

So $f(x + h) = 5$

Now $f'(x) =$

$=$

$=$

As ,

4.1 Differentiation from first principles

You try

Differentiate, from first principles, $y = 2x^2 - x + 1$

Let $f(x) = 2x^2 - x + 1$

So $f(x + h) = 2(x + h)^2 - (x + h) + 1$

Now $f'(x) =$

=

=

=

=

=

As ,